Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 9 / 0 1 R	Signature	

Paper Reference(s)

6669/01R

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 23 June 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. Solve the equation

$$5\tanh x + 7 = 5\operatorname{sech} x$$

Give each answer in the form $\ln k$ where k is a rational number.

(5)

Leave blank

2.

$$9x^2 + 6x + 5 \equiv a(x + b)^2 + c$$

(a) Find the values of the constants a, b and c.

(3)

Hence, or otherwise, find

(b) $\displaystyle\int \frac{1}{9x^2 + 6x + 5}\,\mathrm{d}x$

(2)

(c) $\displaystyle\int \frac{1}{\sqrt{9x^2 + 6x + 5}}\,\mathrm{d}x$

(2)

Leave blank

Question 2 continued

Leave blank

3. The curve C has equation

$$y = \frac{1}{2}\ln(\coth x), \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = -\operatorname{cosech} 2x$$

(3)

The points A and B lie on C.

The x coordinates of A and B are $\ln 2$ and $\ln 3$ respectively.

(b) Find the length of the arc AB, giving your answer in the form $p \ln q$, where p and q are rational numbers.

(6)

Leave blank

Question 3 continued

Leave blank

4.

$$I_n = \int_0^{\sqrt{3}} (3 - x^2)^n \, dx, \quad n \geqslant 0$$

(a) Show that, for $n \geqslant 1$

$$I_n = \frac{6n}{2n+1} I_{n-1}$$

(6)

(b) Hence find the exact value of I_4, giving your answer in the form $k\sqrt{3}$ where k is a rational number to be found.

(5)

Leave blank

Question 4 continued

Leave blank

5. The ellipse E has equation

$$x^2 + 9y^2 = 9$$

The point $P(a\cos\theta, b\sin\theta)$ is a general point on the ellipse E.

(a) Write down the value of a and the value of b.

(1)

The line L is a tangent to E at the point P.

(b) Show that an equation of the line L is given by

$$3y\sin\theta + x\cos\theta = 3$$

(3)

The line L meets the x-axis at the point Q and meets the y-axis at the point R.

(c) Show that the area of the triangle OQR, where O is the origin, is given by

$$k\,\mathrm{cosec}\,2\theta$$

where k is a constant to be found.

(3)

The point M is the midpoint of QR.

(d) Find a cartesian equation of the locus of M, giving your answer in the form $y^2 = \mathrm{f}(x)$.

(4)

Leave blank

Question 5 continued

P 4 4 5 1 3 A 0 1 7 3 2

Leave blank

6. The symmetric matrix **M** has eigenvectors $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

with eigenvalues 5, 2 and –1 respectively.

(a) Find an orthogonal matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{MP} = \mathbf{D}$$

(4)

Given that $\mathbf{P}^{-1} = \mathbf{P}^{\mathrm{T}}$

(b) show that

$$\mathbf{M} = \mathbf{PDP}^{-1}$$

(2)

(c) Hence find the matrix **M**.

(5)

Leave blank

Question 6 continued

P 4 4 5 1 3 A 0 2 1 3 2

Turn over

Leave blank

7. The curve C has equation

$$y = e^{-x}, \qquad x \in \mathbb{R}$$

The part of the curve C between $x = 0$ and $x = \ln 3$ is rotated through 2π radians about the x-axis.

(a) Show that the area S of the curved surface generated is given by

$$S = 2\pi \int_0^{\ln 3} e^{-x} \sqrt{1 + e^{-2x}} \, dx$$

(3)

(b) Use the substitution $e^{-x} = \sinh u$ to show that

$$S = 2\pi \int_{\operatorname{arsinh} \alpha}^{\operatorname{arsinh} \beta} \cosh^2 u \, du$$

where α and β are constants to be determined.

(5)

(c) Show that

$$2\int \cosh^2 u \, du = \frac{1}{2} \sinh 2u + u + k$$

where k is an arbitrary constant.

(2)

(d) Hence find the value of S, giving your answer to 3 decimal places.

(2)

Leave blank

Question 7 continued

Leave blank

8. The plane Π_1 has vector equation $\mathbf{r}.\begin{pmatrix}2\\1\\3\end{pmatrix} = 5$

The plane Π_2 has vector equation $\mathbf{r}.\begin{pmatrix}-1\\2\\4\end{pmatrix} = 7$

(a) Find a vector equation for the line of intersection of Π_1 and Π_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.
(6)

The plane Π_3 has cartesian equation

$$x - y + 2z = 31$$

(b) Using your answer to part (a), or otherwise, find the coordinates of the point of intersection of the planes Π_1, Π_2 and Π_3
(3)

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Question 8 continued

Q8

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END